

## COMPLEX TRACE ANALYSIS OF SEISMIC SIGNAL BY HILBERT TRANSFORM

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### ABSTRACT

Non-Stationary statistical Geophysical Seismic Signal Processing (GSSP) is of paramount importance for imaging underground geological structures and is being used all over the world to search for petroleum deposits and to probe the deeper portions of the earth. Hilbert Transform can be made fashionable for seismic signal processing with a view to precise subsurface imaging. The complex trace attributes are widely used in seismic interpretation. These attributes are based on the properties of the analytic signal. The analytic signal is usually computed through the Fourier transform. This transform has a global character and hence is not fit for characterization of local signal parameters. The complex trace attributes are derived from the analytic signal. The most commonly used attributes are the envelope or reflection strength, the instantaneous phase, and the instantaneous frequency, the derivative of the phase. The analytic signal is usually computed through the sum of the real-valued trace with its imaginary Hilbert transform. Since the Hilbert transform is a step function in frequency, it has a global character in time. This behavior can cause undesirable Gibbs phenomena. Due to these phenomena, the characterization of seismic signals through the analytic signal is often contaminated with noise and therefore scientists have exploited different approaches for the computation of these attributes. The analytic signal that consists of the real-valued signal with this so-called local Hilbert transform has superior characterization qualities compared to the conventional Fourier-based analytic signal. Due to its smoothness in frequency and its finite length in time, the complex trace attributes computed through convolution with these filters tend to be very accurate. The performance of transform depends on the filter length. The Hilbert transform preserves the orthogonality, local smoothness, and connection (coefficients) of a scaling function (wavelet) basis. In other words the Hilbert transform of a wavelet is a wavelet. To describe a signal simultaneously in time and in frequency is to consider its instantaneous frequency. The signal and its Hilbert Transform are orthogonal. This is because by rotating the signal  $90^\circ$  we have now made it orthogonal to the original signal, that being the definition of orthogonality. The signal and its Hilbert Transform have identical energy because phase shift does not change the energy of the signal only amplitude changes can do that. In Digital Signal Processing we often need to look at relationships between real and imaginary parts of a complex signal. These relationships are generally described by Hilbert transforms. Hilbert transform not only helps us relate the real and imaginary components but it is also used to create a special class of causal signals called analytic which are especially important in simulation. The sweetness factor is defined as the quotient of instantaneous amplitude to instantaneous frequency of a seismic trace which are determined by Hilbert Transform. HT limitation is that it does not provide physically meaningful results in situations where the seismic trace contains events that overlap in time but have different frequencies. Such cases are frequently encountered in exploration seismology. This limitation is rectified by S-Transformation.

### INTRODUCTION

Geological interpretation of seismic data is commonly done by analyzing patterns of seismic amplitude, phase and frequency in map and section views across a prospect area. Although many seismic attributes have been utilized to emphasize geologic targets and to define critical rock and fluid properties, these three simple attributes – amplitude, phase and frequency – remain the mainstay of geological interpretation of seismic data.

Any procedure that extracts and displays any of these seismic parameters in a convenient and understandable manner is an invaluable interpretation tool. That Hilbert transform approach now forms the basis by which almost all amplitude, phase and frequency attributes are calculated by today's seismic interpretation software.

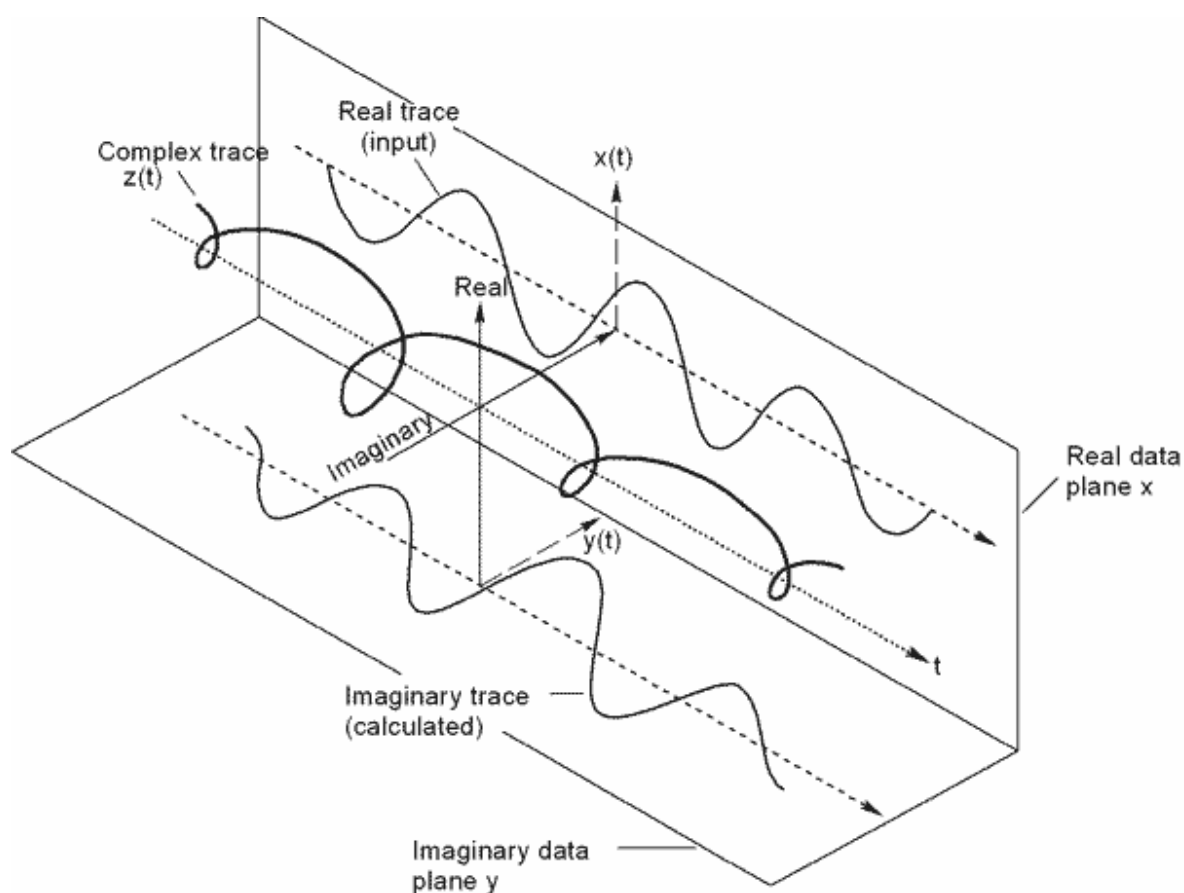


Fig 1. - A complex seismic trace consisting of a real part  $x(t)$ , which is the actual seismic trace, and an imaginary part  $y(t)$ , which is a mathematical function calculated from the real part by a Hilbert transform. When the real and imaginary parts are added in a vector sense, the result is a helical spiral centered on the seismic time axis ( $t$ ). This helical trace is the complex seismic trace.

### The Complex Seismic Trace

The action of the Hilbert transform is to convert a seismic trace  $x(t)$  into what first appears to be a mysterious complex seismic trace  $z(t)$  as shown on figure 1

In this context, the term “complex” is used in its mathematical sense, meaning it refers to a number that has a real part and an imaginary part. The term does not imply that the data are difficult to understand.

This complex trace consists of the real seismic trace  $x(t)$  and an imaginary seismic trace  $y(t)$  that is the Hilbert transform of  $x(t)$ .

On figure 1 these two traces are shown in a three-dimensional data space  $(x, y, t)$ , where  $t$  is seismic time,  $x$  is the real-data plane, and  $y$  is the imaginary-data plane. The actual seismic trace is confined to the real-data plane; the Hilbert transform trace is restricted to the imaginary-data plane.

These two traces combine to form a complex trace  $z(t)$ , which appears as a helix that spirals around the time axis.

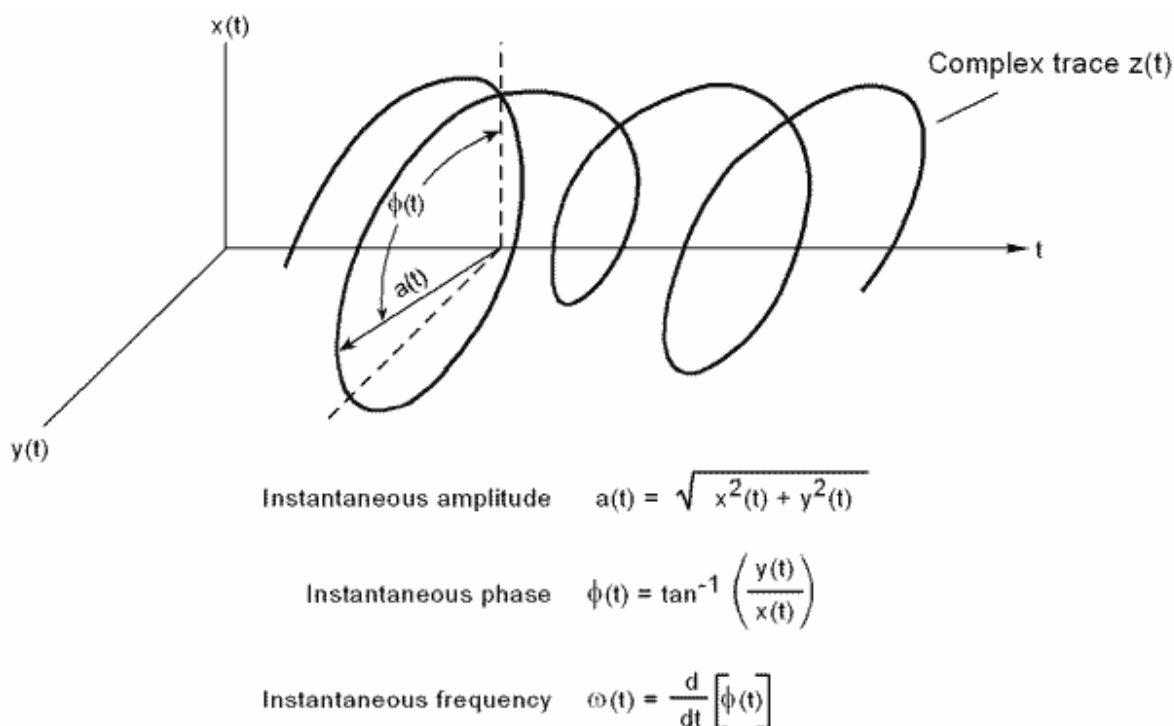


Fig 2. - Instantaneous seismic attributes – amplitude  $a(t)$ , phase  $\phi(t)$  and frequency  $\omega(t)$  – that can be calculated from a complex seismic trace using the listed equations

The projection of complex trace  $z(t)$  onto the real plane is the actual seismic trace  $x(t)$ ; the projection of  $z(t)$  onto the imaginary plane is the Hilbert transform trace  $y(t)$ . At any coordinate on the time axis, a vector  $a(t)$  can be calculated that extends perpendicularly away from the time axis to intercept the helical complex trace  $z(t)$  as shown on figure 2. The length of this vector is the amplitude of the complex trace at that particular instant in time – hence the term “instantaneous amplitude.”

### Physical Meaning Of Instantaneous Attributes

Seismic interpreters know that frequency spectrum usually becomes lower with increasing arrival time as the high-frequency components are attenuated faster than low-frequency components. The way to deal with these non-stationary processes is to Fourier transform finite time windows, which are continuously shifted in time. The results tell us how the frequency components evolve with time. However, the time window must have a finite width. The time window should be large enough to make

Fourier transform meaningful. This introduces a resolution problem since there is a tradeoff between the length of time window and the accuracy of frequency estimation. Also, the time window, although very small, may not satisfy the assumption that the process within it is stationary. Hence, it is useful to have a theoretical approach to cope with the continuously varying frequency spectrum in order to define instantaneous attributes.

Conventional seismic attributes in time-lapse seismic surveys are based on a description of the real seismic trace. Event picks on the top and bottom of a reservoir and subsequent calculation of time shift and amplitude variation may be difficult or inaccurate due to interference. This resolution limit could lead to misinterpretation of amplitude change. The Fourier transform in a time window allows us to look at its average properties, but it does not examine local variations, which may change in response to wavelet interference. In addition, Fourier analysis, assuming that all spectral components are stationary in time, is not effective for analysis of nonstationary seismic signals.

## CONCLUSIONS

Complex trace analysis enables separation of amplitude from angle (frequency and phase) and definition of instantaneous amplitude, phase and frequency, which are independent of each other. The instantaneous amplitude, a direct response to the magnitude of reflectivity, defines single lobes for individual wavelets and thus has more power to resolve reflectors than the conventional seismic trace, which combines amplitude and angle. The instantaneous frequency is a measure of most energy loaded frequency or center frequency of the instantaneous power spectrum and, along with the instantaneous phase, traces frequency and phase change with time. It is also sensitive to wavelet interference, manifesting abrupt change at strong interference. In the case of strongly overlapping wavelets, instantaneous amplitude and frequency have characteristics that help identify wave interference. In time-lapse seismic surveys, the power of resolution improves event picks and calculation of time shift and amplitude variation, and the representation of frequency and phase facilitates the study of frequency and phase change with time.

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